Abitur 2008 Aufgabe 2 (Geometrie) Lösung:

1. $\overrightarrow{AB}\times\overrightarrow{AM} = 16\begin{pmatrix}2\\1\\0\end{pmatrix}\times\begin{pmatrix}13\\14\\-3\end{pmatrix} = 16\begin{pmatrix}-3\\6\\15\end{pmatrix} = 48\begin{pmatrix}-1\\2\\5\end{pmatrix} \Rightarrow e: \begin{pmatrix}-1\\2\\5\end{pmatrix}\cdot\left(\vec{x}-\begin{pmatrix}6\\-12\\22\end{pmatrix}\right) = 0$

$e: -x+2y+5z-(-6-24+110) = 0 \Leftrightarrow e: -x+2y+5z-80 = 0$

2. $\vec{c} = \vec{m}+\overrightarrow{AM} = \begin{pmatrix}19\\2\\19\end{pmatrix}+\begin{pmatrix}13\\14\\-3\end{pmatrix} = \begin{pmatrix}32\\16\\16\end{pmatrix} \Rightarrow C(32|16|16)$

$\vec{d} = \vec{c}-\overrightarrow{AB} = \begin{pmatrix}32\\16\\16\end{pmatrix}-\begin{pmatrix}32\\16\\0\end{pmatrix} = \begin{pmatrix}0\\0\\16\end{pmatrix} \Rightarrow D(0|0|16)$

$\overrightarrow{DC} = \begin{pmatrix}32\\16\\0\end{pmatrix} = \overrightarrow{AB} \Rightarrow \text{PG} \quad \overrightarrow{AB}\cdot\overrightarrow{AD} = 16\begin{pmatrix}2\\1\\0\end{pmatrix}\cdot\begin{pmatrix}-6\\12\\-6\end{pmatrix} = 16\cdot(-12+12+0) = 0 \Rightarrow \perp \Rightarrow \text{Rechteck}$

3. $x_1x_2-\text{Ebene}: x_3 = 0 \text{ in } e_1 \Rightarrow 2x_1-4x_2+0 = 65 \Rightarrow x_1 = 2x_2+\frac{65}{2} \Rightarrow \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}2x_2+65/2\\x_2\\0\end{pmatrix} =$

$\begin{pmatrix}65/2\\0\\0\end{pmatrix}+x_2\begin{pmatrix}2\\1\\0\end{pmatrix} \Rightarrow s_{x_1x_2}: \vec{x} = \begin{pmatrix}65/2\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}$

4. 1 1) $\vec{n}_{\text{tribüne}}\begin{pmatrix}2\\-4\\5\end{pmatrix} \quad \vec{n}_{\text{Boden}} = \begin{pmatrix}0\\0\\1\end{pmatrix} \Rightarrow \cos(\alpha) = \dfrac{\left|\begin{pmatrix}2\\-4\\5\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right|}{\sqrt{4+16+25}\cdot 1} = \dfrac{5}{\sqrt{45}} \approx 0,745 \Rightarrow \alpha = 41,81^{o}$

2) $\cos(\beta) = \dfrac{\overrightarrow{BS_2}\cdot\overrightarrow{BC}}{\left|\overrightarrow{BS_2}\right|\left|\overrightarrow{BC}\right|} = \dfrac{\begin{pmatrix}-6\\12\\3\end{pmatrix}\begin{pmatrix}-6\\12\\-6\end{pmatrix}}{\sqrt{36+144+9}\ \sqrt{36+144+36}} = \dfrac{36+144-18}{13,74\cdot 14,69} = \dfrac{162}{202,05} = 0,802 \Rightarrow \beta = 36,69^{o}$

4.2 $d(M,e_1) = \frac{1}{\sqrt{45}}|(2\cdot 19-4\cdot 2+5\cdot 19-65| = \frac{60}{\sqrt{45}} \approx 8,94 < 10$ Vorschrift erfüllt.

4.3 $g: \vec{x} = \begin{pmatrix}6\\-12\\22\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-19\end{pmatrix} \quad \{;A'\} = g\cap e_1: \quad 2(6-\lambda)-4(-12+2\lambda)+5(22-19\lambda) = 65$

$\Rightarrow 12-2\lambda+48-8\lambda+110-95\lambda = 65 \Rightarrow -105\lambda = -105 \Rightarrow \lambda = 1 \Rightarrow \overrightarrow{OA'} = \begin{pmatrix}6\\-12\\22\end{pmatrix}+\begin{pmatrix}-1\\2\\-19\end{pmatrix}$

$= \begin{pmatrix}5\\-10\\3\end{pmatrix} \Rightarrow A'(5|-10|3)$

4.4 $g_{S_1D}: \vec{x} = \begin{pmatrix}0\\0\\16\end{pmatrix}+\lambda\begin{pmatrix}0\\0\\1\end{pmatrix} \Leftrightarrow x_3 = 16+\lambda \ \{E\} = e_1\cap g_{S_1D}: 2\cdot 0-4\cdot 0+5\cdot(16+\lambda) = 65$

$\Rightarrow 5\lambda = -15 \Rightarrow \lambda = -3 \Rightarrow E(0|0|13)$ entsprechend $F(32|16|13)$

$\overrightarrow{DE} = \begin{pmatrix}0\\0\\13\end{pmatrix}-\begin{pmatrix}0\\0\\16\end{pmatrix} = \begin{pmatrix}0\\0\\-3\end{pmatrix} \quad \overrightarrow{DC} = \begin{pmatrix}32\\32\\16\end{pmatrix}-\begin{pmatrix}0\\0\\16\end{pmatrix} = \begin{pmatrix}32\\32\\0\end{pmatrix} \Rightarrow \mu = \left|\overrightarrow{DE}\right|\cdot\left|\overrightarrow{DC}\right| =$

$= 3\cdot 32\cdot\sqrt{2} = 96\sqrt{2}\ m^2$