A Mathematical Outline of a Theory of Matrial Needs

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In history the concept of need is frequently used as a basic concept as well in philosophy as in psychology or biology. But nowhere you can find a philosophically satisfying description or even definition.
So in this part we want to find a precise but open definition of the matrial concept of a need and then analyze the important problem of how a set of needs can be satisfied.
Let us try to achieve this mathematization in a simplified way.

Def.1: Let E be a class of non-empty sets.
Let $\Sigma \subseteq \wp(\cup E)$ Denote a **family of situations (of first order)** of E, with the following axioms:

(S1) $\emptyset \in \Sigma$
(S2) $E \subseteq \Sigma$
(S3) $\sigma_1, \sigma_2 \in \Sigma \Rightarrow \sigma_1 \cap \sigma_2 \in \Sigma$
(S4) $\sigma_1, \sigma_2 \in \Sigma \Rightarrow \sigma_1 \cup \sigma_2 \in \Sigma$
(S5) $\sigma \in \Sigma \Rightarrow \overline{\sigma} \in \Sigma, \quad \overline{\sigma} := \cup E \setminus \sigma$

Let us call E the class of **elementary situations**[1].
In an analogous way we can define a **family of situations** $\Sigma^{(n)}$ **of the order n.**
$\Sigma^* := \Sigma^{(2)}$. Let E be the union of the subclass Γ, the class of the **need situations**, and Π, the class of the **satisfaction situation**s: $E = \Gamma \cup \Pi$.

Def.2: Let $\epsilon_1, \ldots, \epsilon_n \in E$ and $\forall_{\nu \in \{1,\ldots,n\}} e_\nu := \bigcap_{i \in \{1,\ldots,\nu\}} \epsilon_i \in \Sigma \setminus \{\emptyset\}$
Let us call the set $e^n := e := \{e_1, \ldots, e_n\} \in \Sigma^*$ the **situation type of** $\epsilon_i (i=1,\ldots,n)$ and e_i the stages of e^n.
Let us call $M(e^n) := \{\epsilon_1, \ldots, \epsilon_n\} \in \Sigma^*$ the **historic matrix of** e^n.

Def.3: Let us call $\epsilon \in E$ **matrization** or **topization** of e^n or ***e^n-situation*** [2]: $\Leftrightarrow e_n \subseteq \epsilon$
(better than realization or actualization, as those are too special).

Let us call a situation type $a^n \in \Sigma^*$ **matricizable in** $\epsilon \in E \setminus M(a^n)$ or in formal notation $a^n < \epsilon \ : \ \Leftrightarrow \ a_n \cap \epsilon \neq \emptyset$.
Let us call a^n **matricizable**: $\Leftrightarrow \exists_{\epsilon \in E \setminus M(a^n)} a_n \cap \epsilon \neq \emptyset$

1 The $\epsilon - situations$ are, as it were, the mental „bricks", the mental cells which are getting an inner structure as they progress – except for the nucleus, the residue of feeling.
Instead of the mathematical intersection and union, another new relation could perhaps be more adequate.
2 An amplification of the concept of situation type will be necessary, if there are *kinds* of situations in the form of $e_\nu \cup \cap \epsilon_i$ without a continuos residue of feeling.
This may correspond to *Wittgensteins* „family similarity" (cf. PU 67). A situation type, or its amplification, may represent what we call a concept or a pre-object of one or more persons. The type is produced through its historic matricizations. But how can we know that a situation is a matricization of a type? Only because it is to be, because there is an unarticulated need feeling. In progressing and fixing the type, the croterion will emancipate from the need into a simple knowledgem yet without reasons.

Def.4: Let us call a situation type $a^n \in \Sigma^*$ **fixed**: $\Leftrightarrow \exists_{k<n}\ a_k = a_{k+1} = \ldots = a_n = a_{n+1}$

Def.5: Let us call a situation type $b^m \in \Sigma^*$ a **specialization** of the situation type $a^n \in \Sigma *$ **(in formal notation** $b^m \prec a^n$): $\Leftrightarrow M(b^m) \subseteq M(a^n)$
a^n then would be called a **generalization** of b^m .

Prop.1: **1)** If $a^n \in \Sigma *$ is a fixed type, then it follows: $a^n < \epsilon \Leftrightarrow a_n \subseteq \epsilon$

2) $b^m \prec a^n \Rightarrow a_n \subseteq b_m$

3) Let $a^n, b^m \in \Sigma *$: $a^n < \epsilon \wedge b^m \prec a^n \Rightarrow b^m < \epsilon$

4) Let $a^n, b^m \in \Sigma * \wedge b^m$ fixed: $b^m < \epsilon \wedge b^m \prec a^n \Rightarrow a^n < \epsilon$

5) Let $a^n, b^m \in \Sigma * \wedge b^m$ fixed $\wedge b^m \prec a^n$: $a^n < \epsilon \Leftrightarrow b^m < \epsilon$

Proof: 1) $a^n > \epsilon \Rightarrow a_n \cap \epsilon \neq \emptyset \Rightarrow a_{n+1} = a_n \cap \epsilon$; a^n is fixed, therefore
$a_{n+1} = a_n \Rightarrow a_n \subseteq \epsilon$

2) $b^m \prec a^n \Rightarrow \{\epsilon_{i_1}, \ldots, \epsilon_{i_m}\} := M(b^m) \subseteq M(a^n) =: \{\epsilon_1, \ldots, \epsilon_n\} \Rightarrow$
$a_n = \epsilon_1 \cap \ldots \cap \epsilon_n \subseteq \epsilon_{i_n} \cap \ldots \cap \epsilon_{i_m} = b_m$

3) $b^m \prec a^n \Rightarrow a_n \subseteq b_m$
Let $a^n < \epsilon \Rightarrow a_n \cap \epsilon \neq \emptyset$, as $a_n \subseteq b_m \Rightarrow b_m \cap \epsilon \neq \emptyset \Rightarrow b^m < \epsilon$

4) Let $b^m < \epsilon \Rightarrow b_m \cap \epsilon \neq \emptyset$ and as b^m is fixed , $b_m \subseteq \epsilon$
$a_n \subseteq b_m$ hence $a_n \subseteq \epsilon$ or $a_n \cap \epsilon \neq \emptyset$ or finally $a^n < \epsilon$

Def.6: Let $\gamma_i \in \Gamma$ and let $g_r := \gamma_1 \cap \ldots \cap \gamma_r$ be the feeling of need and $g^r := \{g_{1,\ldots}, g_r\}$ its type, $p_m := \pi_1 \cap \ldots \cap \pi_m$ the feeling of satisfaction and $p^m := \{p_{1,\ldots}, p_m\}$ the respective type.
Then let us call $n: p_m \rightarrow g_r$ the **need function** and $n(p_m) = g_r$ **the need for p**.

Example: The type of feeling *hunger* intends the type of feeling, the fact *eating,* i.e. *hunger* is the need to *eat.*

Thus we have defined, as precisely as possible, and in relation to the perspective of this paper, a feeling of a need as a need to do something or for something. However, this definition is typically historical, i.e. In the course of time it may become more concrete, fixed or varied. At this time, the definition has been chosen to be the starting point for a „need logic“; however, we do not want to develop this further here.
Instead let us deal with the problem of how to satisfy a set $N = \{n^{1,\ldots}, n^r\}$ of needs.

First Case: There is at least one $n \in N$ which is not satisfiable, even if it dies not conflict with others. There are at least two subcases of this:

a) Let us call **Don-Juan-needs** those needs, the satisfation of which only seems to be real, as it immediately reproduces the same deficiency again (cf. *Adorno*). Their origin, in terms of need theory (not their political or economic origin) obviously lies in the fact that the need has not been articulated or cannot be articulated as „need of".

Things are similar with *Goethe's* „Faust": Faust is driven on and on; all his attempts to interpret his deficiency are generalized instead of concretized in that last sentence saying the female principle would make Man „ascend".

It is a similar situation as which a child's ennui which indeed implies a need; however, the child cannot find the articulation (action) to satisfy it. Here we touch the aspect of transcendence (cf. *Parmenides'* goddess) which deals with deficiency in general and tries to overcome it.

We would like to understand these Don-Juan-needs as precise manifestations of **false matrial needs of the first kind.**

b) Let us now proceed to the definition abstracted from the emotional component for the time being[3].
Let us call a need $n(p_m)$ **satisfiable**, if its type p^m is matricizable.
Needs shall be called **metabolical***,* if their satisfaction by a certain type produces a need for the respective anti-type; i.e. The need turns to the opposite. This is understood here as a **false matrial need of the second kind**. Needs like that arte characterized by the fact that they do not release you any more, neither in the positive sense (type) nor in the negative sense (antitype) so that you lose your freedom, you are permanently pressed on by both types.

Def.7: A need n(p) shall be called **metabolical***:* $\Leftrightarrow n(p)<\epsilon \;\Rightarrow\; n(p^{\sim})<\epsilon$ [4]

At this point we introduce two further axioms:

Separation axiom (SA): $n(p)<\epsilon \;\Rightarrow\; \neg p<\epsilon$

i.e. the occurance of a need for p implies the non-occurence of p.

Exclusion axiom (EA): $p<\epsilon \;\vee\; p^{\sim}<\epsilon$

i.e. there is either a type p or the antitype p˜ for all constituted p of satisfaction situations.[5]

Prop.2: **(1)** $n(p)$ metabolic $\Rightarrow n(p^{\sim})<\epsilon \;\vee\; n(p)<\epsilon$

3 This abstraction might be important in connection with a system of ecological ethics, as realized during a lecture by *D. Birnbacher* recently. Thus it would be possible to include non-feeling beings.

4 $p^{\sim}$ Shall denote the antitype of p.

5 Naturally, the concept of the antitype is rather difficult to define. However, it is not a precondition for p to be intentional.

(2) $n(p)$ metabolic $\Rightarrow n(p^{\sim})$ metabolic

Proof: (1) $\neg(n(p)<\epsilon) \overset{(Def.7)}{\Rightarrow} n(p^{\sim})<\epsilon$

(2) $\neg(n(p^{\sim})<\epsilon) \overset{(Def.7)}{\Rightarrow} n(p)<\epsilon$ (presupposed is that $p^{\sim\sim}=p$)

Prop.3: Let be q a generalization or specialization of p, p and q fixed and n(p) metabolic, then:

$$n(q)<\epsilon \Rightarrow n(p)<\epsilon$$

Proof: $n(q)<\epsilon \overset{(SA)}{\Rightarrow} \neg(q<\epsilon) \overset{Prop.1(5)}{\Rightarrow} \neg(p<\epsilon) \overset{(EA)}{\Rightarrow} p^{\sim}<\epsilon \overset{(SA)}{\Rightarrow} \neg(n(p^{\sim})<\epsilon) \overset{Prop.2(2)\wedge Def.7}{\Rightarrow}$
$\Rightarrow n(p)<\epsilon$

This proposition means that you cannot escape from fixed metabolic needs through specialization or generalization. Such needs may be characterized as *diabolical.* Conflicts with metabolical needs are especially critical (see below).

Second Case: Let us consider all needs of N as satisfiable, isolated from all other needs.

Def.8: A set N of needs shall be called **(diachronically) satisfiable**, if there exists a future sequence of satisfaction situations in which all needs of N are matricizable. If the sequence consists only of one member, N shall be called **synchronically satisfiable**.

Let E_{dt} denote the class of all „hypothetical elementary situations“ , i.e. The satisfaction situations existing in a relatively short future period dt, including the present time, and let $B(n_i)$ be the class of satisfaction situations of n_i from E_{dt} .
Then N is synchronically satisfiable, if $\bigcap_i B(n_i)\neq\emptyset$.

The opposite case is the critical one. A potential degree of difficulty may be roughly expressed by the following definition:

Def.9: Let M be a class of sets, $i(M)$ the class of all **isolated sets of M**, i.e. Such sets of M the intersections of which with other sets of M are empty.
Let $a(M):=M\setminus i(M)$ the class of the **associated sets of M**
and $d(a(M)):=\{M_1\cap M_2 \ / \ M_1, M_2\in a(M)\}\setminus\{\emptyset\}$ the class of all non-empty intersections in $a(M)$. The mapping $m:M\rightarrow i(M)\cup d(a(M))$ shall be called the **minimization of M** and $m(M)$ the **minimized set of M.**

Prop.4: Let M be a finite set. There exists a number $p\in\mathbb{N}$ so that $m^p(M)$ is a class of isolated sets: $\forall_{n\in\mathbb{N}} m^{p+n}(M)=m^p(M)$.

Def.10: The smallest number q of that kind shall be called the **order of M** and $m^q(M)$ the **minimal system of M**.

Prop.5: For each finite set M there exists one and only one minimal system of M.

If the minimal system of $B=\{B(n) \ / \ n\in\mathbb{N}\}$ contains only one element, then N is synchronically satisfiable. The number of elements of $m^q(B)$ can be used to measure the problems which may arise through the satisfaction of conflicting needs of N. If $|m^q(B)|>1$, then it could happen that the satisfaction of one part of the needs of N contradicts the satisfaction of another part, and this means the needs cannot be satisfied independent of each other.

For example, a certain satisfaction of the need of quick exploitation of energy resources might, in the long run, contradict the satisfaction of the need of better health.

The concept of „contradictional cycles" is characteristic of the theoretical solution of these problems.

Def.11:[6] a^n shall **contradict** b^m in $\epsilon \in E$ (in formal notation: $a^n \dashv_\varepsilon b^m$): $\Leftrightarrow$
$a_n \subseteq \epsilon \wedge \underset{\epsilon' \in E \setminus (M(a^n) \cup M(b^m))}{\exists} \neg(b^m < \epsilon')$

$$a^n \dashv b^m \quad :\Leftrightarrow \underset{\epsilon \in E}{\exists} \quad a^n \dashv_\varepsilon b^m$$

$$n(a^n) \dashv_\varepsilon n(b^m) \quad :\Leftrightarrow a^n \dashv_\varepsilon b^m$$

$$n(a^n) \dashv n(b^m) \quad :\Leftrightarrow a^n \dashv b^m$$

Def.12: k needs $n_1, \ldots, n_k$ shall form a **k-contradictional cycle (k-CC)** $<n_1, \ldots, n_k>$ **,** if $n_1 \dashv n_2 \dashv \ldots \dashv n_k \dashv n_1$

To be able to formulate and prove simply the main proposition we want to put ahead a few more definitions and lemmata.

Def.13: **1)** A subset $G_\perp$ of $N^2 = \{n_1, \ldots, n_r\} \times \{n_1, \ldots, n_r\}$ shall be called a **(contradictional) graph** on N: $\Leftrightarrow (n, m) \in G_\perp \Leftrightarrow n \dashv m$

2) A graph $G_\perp$ is called **irreflexive** $:\Leftrightarrow \underset{n \in N}{\forall} (n, n) \notin G_\perp$

3) A graph $G_\perp$ is called **complete** $:\Leftrightarrow \underset{(n,m) \in N^2}{\forall} (n, m) \in G_\perp \vee (m, n) \in G_\perp$

4) A graph $G_\perp$ is called **k-cycle-free** $:\Leftrightarrow$ $G_\perp$ does not possess a k-CC.

5) A graph $G_\perp$ is called **cycle-free** $:\Leftrightarrow$ $G_\perp$ is k-cycle-free for all k with $1 \leq k \leq r$

6) A need $n \in N$ is called **initial element of** $G_\perp$ $:\Leftrightarrow$ n has no antecessor, i.e. $\neg(\underset{m \in N}{\exists})(m, n) \in G_\perp$

7) A need $n \in N$ is called **final element of** $G_\perp$ $:\Leftrightarrow$ n has no successor, i.e. $\neg(\underset{m \in N}{\exists})(n, m) \in G_\perp$

6 This is one of the weakest forms of contradicting.

Def.14: **1)** Let be $N=\{n_1,\dots,n_r\}$ and $G_\perp$ a graph on N.
A permutation $(n_{i_1},\dots,n_{i_r})$ of N shall be called a **matrization-order** or **M-order of** $G_\perp$ if and only if behind n_{i_ν} there is no n_{i_μ} in $G_\perp$, i.e. $:\Leftrightarrow$
$\underset{\nu}{\forall}(1\le\nu\le r\rightarrow(\underset{\mu}{\forall}\nu<\mu\le r\rightarrow(n_{i_\nu},n_{i_\mu})\notin G_\perp))$

2) Let us call N **diachronically satisfiable**, if there exists a M-order of $G_\perp$ [7]

Lemma 1: Let be $G_\perp$ an non-empty graph on N.
$G_\perp$ is cycle-free $\Leftrightarrow$ N posseses final (initial) elements of $G_\perp$

Proof: Suppose there would be no final elements. Because $G_\perp$ is not empty, there exists an element $(n_{i_1},n_{i_2})\in G_\perp$. As $G_\perp$ is cycle-free, $n_{i_2}\neq n_{i_1}$: $n_{i_1} \dashv n_{i_2}$
n_{i_2} is not final, hence there exists a need $n_{i_3}\in N$ with $(n_{i_2},n_{i_3})\in G_\perp$ (and $n_{i_3}\neq n_{i_1}$ and $n_{i_3}\neq n_{i_2}$ because $G_\perp$ is cycle-free): $n_{i_1} \dashv n_{i_2} \dashv n_{i_3}$. n_{i_3} is not final, hence there exists a need $n_{i_4}\in N$ with $(n_{i_3},n_{i_4})\in G_\perp$ (and $n_{i_4}\neq n_{i_3}$ and $n_{i_4}\neq n_{i_2}$ and $n_{i_4}\neq n_{i_1}$, because $G_\perp$ is cycle-free): $n_{i_1} \dashv n_{i_2} \dashv n_{i_3} \dashv n_{i_4}$; and so on until all elements of N are used (N is finite). For the last element n_{i_r} there must be in this line a next one. But this is not possible, because $G_\perp$ is cycle-free. The proof for the initial element is completely analogous.

Lemma 2: Each subgraph of a cycle-free graph is again cycle-free.

Proof: Let $H_\perp$ be a subgraph of $G_\perp$. If $H_\perp=\emptyset$ then $H_\perp$ is cycle-free. Let now be $H_\perp\neq\emptyset$ and $<n_{i_1},\dots,n_{i_s}>$ a cycle in $H_\perp$. Then $<n_{i_1},\dots,n_{i_s}>$ is a cycle in $G_\perp$ which states a contradiction to the precondition.

Lemma 3: Let be $G_\perp$ a non-empty graph on N.
If $G_\perp$ is cycle-free, then N possesses a M-order.

Proof: $G_\perp$ cycle-free $\overset{Lemma 1}{\Rightarrow}$ N possesses final elements. For $A\subseteq N$ let $F(A)$ be the class of all final elements of A.
$N_1:=N\backslash F(N)$ $|N_1|<|N|$, because $G_\perp$ is cycle-free, $G_{\perp 1}=G_\perp\cap N_1^2$ is a cycle-free subgraph according to Lemma 2 $\Rightarrow$ N_1 possesses final elements according to Lemma 1.
$N_2:=N_1\backslash F(N_1)$ $|N_2|<|N_1|$, $G_{\perp 2}=G_\perp\cap N_2^2$ is again a cycle-free subgraph according to Lemma 2 $\Rightarrow$ N_2 possesses final elements according to Lemma 1. And so on unto $N_s:=N_{s-1}\backslash F(N_{s-1})$ $|N_s|<|N_{s-1}|$, $G_{\perp s}=G_\perp\cap N_s^2$ is a cycle-free subgraph according to Lemma 2. Since N is finite at some point this chain is ending. Let N_s be the last element of this chain with $N_s\neq\emptyset$ (then holds: $N_{s+1}=\emptyset$).
Let be $F(N_i)$ with $i=1,\dots,s$ in any order and $(F(N_i))$ this order. Then $((F(N)),(F(N_1)),\dots,(F(N_s)))$ is a M-order:
[Let $k\in\{0,1,\dots,s\}$ and $N_o:=N$. All elements of $F(N_k)$ are final elements of N_k for $G_{\perp k}=G_\perp\cap N_k^2$. They shall be ordered as $(n_{k_1},\dots,n_{k_k})$, where no element is a

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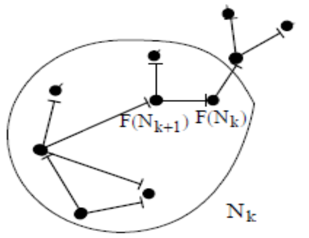
7 If the contradictional graph on N is empty, then each permutation of N will be a M-order and hence N will be diachronically satisfiable.

successor of another of this k-tuple with regard to $G_{\perp k}$, because all are final elements in $G_{\perp k}$. None of these has a successor of $F(N_\rho)$ for $\rho > k$, because $F(N_\rho) \subset N_k, \rho > k$. It is:



$$\begin{array}{ll} N_k & \supset F(N_k) \\ \cup & \\ N_{k+1} & \supset F(N_{k+1}) \\ \cup & \\ N_{k+2} & \supset F(N_{k+2}) \\ \cup & \\ N_{k+3} & \supset F(N_{k+3}) \\ \cdots & \cdots \end{array}$$

]

Lemma 4: A complete graph $G_\perp$ of N which is 1-, 2- and 3-cycle-free is free of any cycles.

Proof: (indirect): Suppose that $G_\perp$ possesses a cycle. Let M be the set of all cycles, which shall be partitioned into nonempty sets M_k of the existing k-cycles $(k>3)$. Let k_o be the smallest of these k. Let $<n_{i_1}, \dots, n_{i_{ko}}>$ be a k_o-cycle. If $(n_{i_3}, n_{i_1}) \notin G_\perp$, then $<n_{i_1}, n_{i_2}, n_{i_3}>$ would be a 3-cycle. Therefore $(n_{i_3}, n_{i_1}) \in G_\perp \Rightarrow <n_{i_1}, n_{i_3}, \dots, n_{i_{ko}}>$ is a $(k_o - 1)$-cycle. This contradicts the minimality.

Now we can formulate and prove the main theorem:

Prop. 6: A set N of needs[8] is exactly then (historically) satisfiable, if it has no contradictional k-cycles.

Proof: 1) Let N be diachronically satisfiable, then there is a M_order $(n_{j_1}, \dots, n_{j_r})$ of N. Suppose there would be a k-CC $<n_{i_1}, \dots, n_{i_k}>$. Let be n_{j_ρ} the first element of the M-order, which is part of the k-CC: Let n_{i_l} this element of the k-CC. But since n_{i_l}, in cause of the cyclicity, has a successor n_{i_m}, this one must be beyond n_{j_ρ} in cause of Def. 14. This implies a contradiction to the minimality of n_{i_l} of the M-order. Hence there is no k-CC.

2) $G_\perp = \emptyset \Rightarrow$ N possesses a M-order (each permutation of N).
$G_\perp \neq \emptyset \Rightarrow G_\perp$ cycle-free (because N has no k-CC) $\overset{Lemma 3}{\Rightarrow}$ N possesses a M-order.
So we conclude according to Def. 14 2) that N is diachronically satisfiable.

Under certain conditions we can precise the last proposition.

Def. 15: 1) A twin set { n_1, n_2 } of needs shall be called **half-contradictional**, if $n_1 \dashv n_2 \vee n_2 \dashv n_1$

2) A set N of needs shall be called **globally half-contradictional,** if each twin set of N is half-contradictional.

Prop.7: A set N of needs, which is globally half contradictional and has no 1-CC or CC-2 or 3-CC, is (diachronically) satisfiable.

8 It should not be forgotten, that the condition of the second case still holds.

Proof: The graph $G_{\perp}$ of the set N of needs, which satisfies the conditions of Prop.7, is a graph which satisfies the conditions of Lemma 4. This means that $G_{\perp}$ is cycle-free. The rest of the proof cf. the second part of the proof of Prop.6.

Prof. 6 means that a matricization (satisfaction) is only possible, if the CC's are removed. Generally they can be removed through specializations of needs[9]. Specializing does not affect non-contradicting needs, they remain non-contradicting; and contradicting needs remain contradicting, if we generalize, as the following proposition shows.

Prop. 8: If p and q are situation types and $n(p)$ and $n(q)$ types of needs and $\epsilon \in E$, then:

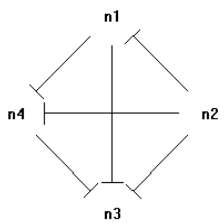
$$p' \prec p \wedge q' \prec q \wedge \neg(n(p) \dashv_{\varepsilon} n(q)) \Rightarrow \neg(n(p') \dashv_{\varepsilon} n(q'))$$

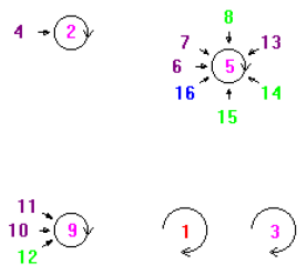
Proof: Let $p'_s \subseteq \epsilon$. $p_n \subseteq p'_s$ include, $p_n \subseteq \epsilon$ and this $\forall_{\epsilon' \in E \setminus (M(p^n) \cup M(q^m))} q^m < \epsilon'$ so that $q_m \cap \epsilon' \neq \emptyset$, and with $q_m \subseteq q'_t$ follows $q'_t \cap \epsilon' \neq \emptyset$ and finally $q'_t < \epsilon'$. Suppose $\epsilon' \in M(p'^s) \cup M(q'^t)$, then $\epsilon' \in M(p'^s) \vee \epsilon' \in M(q'^t)$ or $\epsilon' \in M(p^n) \vee \epsilon' \in M(q^m)$, implying the contradiction of $\neg(\epsilon' \in M(p^n) \cup M(q^m))$.

These specializations mean, among other things, „culturation“. The more conflicts a society has, the stronger are its culturation tendencies.

Proposition 7 means, transferred to a society or to social groups, that globally conflicting groups, whose 2-CC's and 3-CC's are removed are strongly hierarchized because needs can be satisfied only in such a hierarchy.

In the following example we can only matricize in the sequence (n_3, n_4, n_1, n_2) :





We must notice that the person or the need n_3 who/which is hindered most, matricizes first, and that the person or the need n_2 who/which is hindering most, matricizes last („the last will be the first“).

The figure on the right hand side shows the internal dynamics of this set of needs. The numbers indicate the different satisfaction situations: 1 means, that no need is satisfied, the pink colours 2, 3, 5 and 9, that just one need is satisfied, the purple colours 4, 6, 7, 10, 11 and 13, that two and the green colours 8, 12, 14 and 15, that three and 16, that all needs are satisfied.The following table show that more precisely:

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9 Especially if $\epsilon' = \epsilon$, the specialization may have a tendency towards dissociation or „repression“ of the ambivalent situation of satisfaction of n_1 and continuous need of n_2 $(n_1 \dashv n_2)$, thus resulting in a new articulation of n_1 or in a neurosis (cf. *Freud*). Or a need n'_1 which was considered as n_1, could be substituted through articulations by a new family variant.

situation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
satisfied needs	none	n_4	n_3	n_3 n_4	n_2	n_2 n_4	n_2 n_3	n_2 n_3 n_4	n_1	n_1 n_4	n_1 n_3	n_1 n_3 n_4	n_1 n_2	n_1 n_2 n_4	n_1 n_2 n_3	all

Presupposed is in this concrete dynamic that each need tries to be satisfied in each situation („**anapoietic needs**“). It is apparent that the dominant need n_2 (alone satisfied in situation 5) in all situations where it is satisfied, remains the sole winner, n_3 , however, is only isolated "viable". To change however the requirements for the dynamic, for example, that the needs do not try to satisfy themselves on their own (**„indifferent needs“**), so general frustration is evident after the initial situation. The point attractor, then, is the situation of 1.

I would like to focuse now for simplicity on twin and triple groups and discuss the problem here. Let { $n_1, n_{2,} n_3$ } be the set of need in question, that is not satisfiable. This means that there is at least one 2-CC or 3-CC.

First Case: (n_i, n_j) shall be a 2-CC. Such situations are, as it were, „far from equilibrium“, i.e. potentially chaotic, like in puberty, parent-child-conflicts, master-servant-situations, industrial actions, potentia1ly neurotic situations, etc.. In these (non-moral) situations the strongest will follow his way (generalized for large groups: see Haken), i.e. the weaker will be forced to specialize his needs.

Such specialization may imply development, individuation which occur in puberty as well as in Hegel's master-servant-dialectics, in the field of psychology Bateson, with his double-bind concept, and Watzlawik - on a more complex level - have dealt with similar problems.
Things become critical, if a conflict involves a metabolical need, as shown by proposition 3, the specialization of metabolical needs does not result in a solution of the conflict because the need continues to exist, that means it is again the individua1 with non-metabolic needs that must specialize; otherwise the relation will be interrupted.

If both needs are metabolical, there is no chance of a genuine solution of the conflict; the result is that there must either be a therapy or the relation must be broken up.
In a moral context, both persons will -in the case of non-metabolical needs -have to specialize in the same way to solve the conflict, if both needs have „equal rights“. However, this specialization may imply psychological impoverishment, as the respective person may have to give up acquired types. Here lies the conflict between ethical-deontological theories of existentialists and those of Kant's followers. In terms of needs this may mean regression from
tekial to matrial needs.

Second Case: (n_1, n_2, n_3) 3-CC, e.g.:

a) (n_i, n_j) 2-CC: see above.

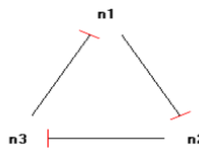
b) $n_i \dashv n_j \wedge \neg(n_j \dashv n_i)$

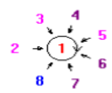
n_j is satisfied earlier than n_i . However, because of the cyclic occurrence of the needs, this is only possible, if the contradiction is limited in time. If the contradiction continues to exist, the satisfaction of n_j becomes difficult. If the respective individuals accept the stable situation { n_i, n_j } the result will be frustration of n_j and, as a consequence, suppression of the need.

Such need suppression was socia1ly sanctioned in Hobbes' and Rousseau's concept of „social submissions“ and the generalization of the act of volition, respectively. If n_j is specialized (to find a solution), this may, in the case of a strong n_i (e.g. generalized will), lead to alienation; thus one would no longer be able to realize one's own will because it would be too much in accordance with society.

In a moral context, both needs would again have to specialize in the same way, in case n_i blocks n_j for a longer period of time. If this is not the case, there is no moral problem.

The categorical imperative of Kant, too, can be seen more clearly and even completed in the light of these reflections. This will not be fo1lowed up here. However, it has to be mentioned a generalization does not work if it is based upon a two-person-model. If we maintain this model, we have reached the border of the moral field, as genuinely social problems (at least of three persons) are not genera1ly solvable in this way, as shown in the diagramme:





The point is that the situation seems to be without any problem to the individual. The sequence of satisfaction for the individual with the need n_1 that does not know the relation between n_2 and n_3, is as follows:
First n_2 is satisfied, then n_1 and then n_3. Thus the individual with n_1 would wait for the individual with n_2 to satisfy first. Because of the symmetry, the situation is analogous for the other individuals. Each individual would wait, and a solution under moral aspects would not be possible. This situation can only be overcome by group communication. Thus a social discussion must take place, in the true sense of the word.

As, with respect to the whole, only minimal specialization is necessary and reasonable which means that not each individua1 has to specialize although each of them has „equal rights“, it would, despite information exchange, not be possible to solve the problem from a formal, categorial point of view. It takes *good will* of one or more individuals to accept asymmetry for a certain time, i.e. to make prior concessions, if necessary.

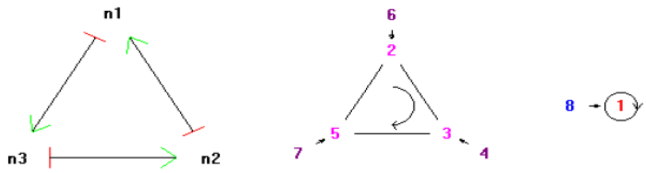
The above mentioned internal dynamics requires the indifferent case.

The table for three needs looks like this:

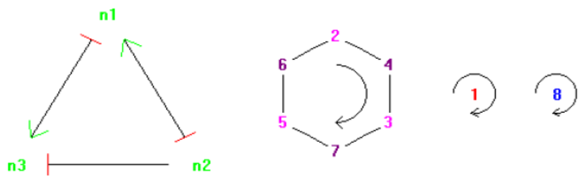
situation	1	2	3	4	5	6	7	8
satisfied needs	none	n_3	n_2	n_2 n_2	n_1	n_1 n_3	n_1 n_2	all

If we consider the case of anapoetic needs we get the following iternal dynamics:

Each situation leads to point attractors, i.e. a need can assert themselves at the expense of others. If they want to matricize all at the same time (situation 8), so the table shows that it would be for all needs the total frustration.
For this case however we found an very interesting configuration: when we add to the contradictional cycle an opposing **„epikourian“[10] anticycle**. The following diagram illustrates this configuration:

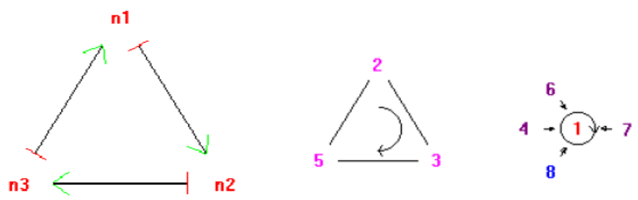


Presupposed are indifferent needs. If we consider again anapoetic needs, we will get an even more satisfying result:



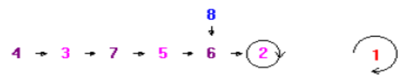
In a two-stroke (or in the indifferent case: in a three-stroke) the needs are regularly satisfied.

The fundamental character is retained even in the case that the needs are individually contradicting themselves (**"cataleptic" needs**)[11]:



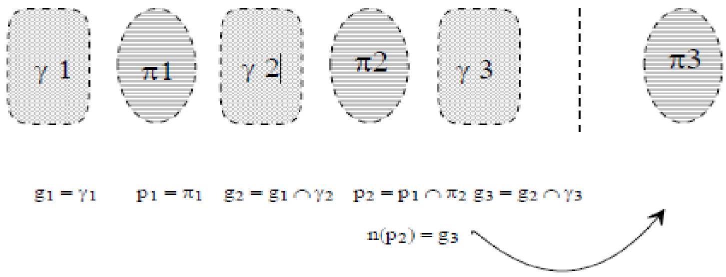
At least in theory this model shows a simple approach to the guiding question that Rousseau had formulated for his social-philosophical problem and which is to this day still not satisfactorily solved[12]. To find a generally acceptable solution in the simplest case of a conflict which requires the social position without suppression or „voluntary“ submission, an epikourean counter-cycle is sufficient. The non-fulfilling of the condition of one of the concerned person in this conflict-situation does not bring any benefit[13] for anybody, because in the next stroke, in the next situation the frustration will be inevitably detected by the other conflict partners:

10 From Gr. επικουρεω: to help, to support.
11 Which is in turn in the event of psychological neurosis, which can be therefore at least temporarily defused just by anti-cycles and then be socially caught.
12 *Rousseau's* main question in: The social contract or principles of constitutional law, Stuttgart 1974: „*The problem is to find a form of association which will defend and protect with the whole common force the person and goods of each associate, and in which each, while uniting himself with all, may still obey himself alone, and remain as free as before.*“. Also *Habermas'* approach in Factuality and Validity, Frankfurt a.M. 1993[3] is not satisfying, even though he clearly states the problem.
13 Cf. *Thomas Hobbes'* strategy of prevention by submission. Even if his argumention force is exemplary, it is worth to analyze more precisely his argumention by a theory of needs to find out his weak points, what I have done in another essay.

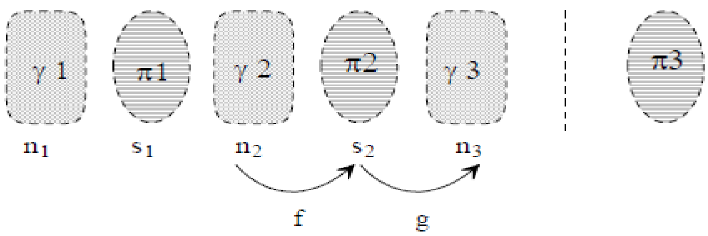


We would like to clarify in this last section the idea of defining needs in a quantified way implied in the philosophical part.

In Def. 6 we have specified the need structurally as the articulation of unease. This articulation was conveyed through the construction of the last type of ease as anticipation of a new feeling of satisfaction.



Now we want to introduce a quantitative aspect. The transition of the need situations n_i to the satisfaction situations s_i can be taken as a function $f: n_i \rightarrow s_i$ as well as the dual transition of the satisfaction situations to the new need situations $g: s_i \rightarrow n_{i+1}$.



We interpret these functions now numerically as ratios of feeling values. We asked a subject to think about any a specific need, and to develop two respective diagrams for f and g that should reflect tubjectively these dependencies. The functions could now be calculated approximately from these diagrams. We took then the composition $g \circ f: n_i \rightarrow n_{i+1}$ which could be interpreted as the representation of the articulation of the type of need achieved at the given stage of development.

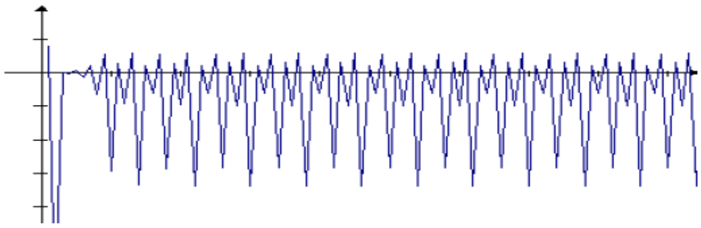
Iterating this composition - maybe with a coefficient of uncertainty - you may get an extrapolated development. Moreover, this composition could even be imagined as "frozen iteration".

We would like to specify two concrete, collected examples.

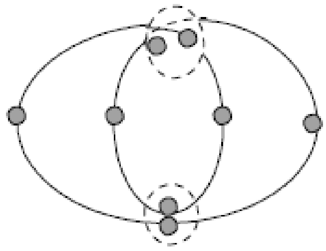
Example 1: For the composition of a 25-year-old male test person was found the following recursion:

$$n_{i+1} = -(n_i \cdot e^{n_i} + 1)^2 + 1$$

The iteration diagram for this recursion is quite interesting:



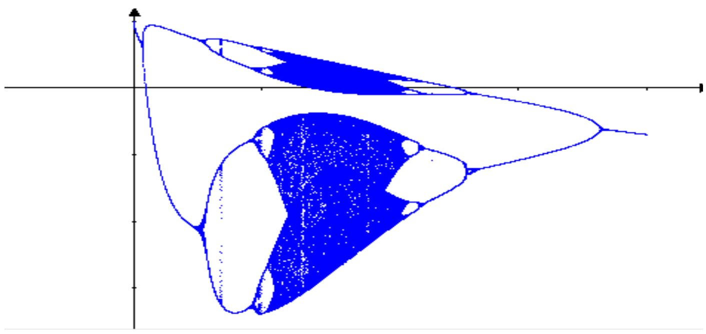
We see here are a limit cycle of 8 values in the order of 0.55 - 0.61, 0.22, - 3,39, 0.6, - 1, 0.3 and -2.85, with two pairs of values (0, 6 / 0, 55) and (0, 3 / 0, 22) very close to one another.



If we interpret values with enough large difference as different articulations of needs, we would only get six different variants. However, since "the" variant (6 / 0,55) has different successors the person would be forced to differentiate the variant (6 / 0,55), which requires a very precise observation of values, lying so close together.

In addition, the iteration diagram is insensibel to a variation of the initial values, it turns very fast again and again to the characteristic values. These properties of extreme precision of perception coupled with the relative indifference of external variations across the system is characteristic of certain mental deseases.

If we generalize the recursion to a family of functions of the form $f_a = -a \cdot (x \cdot e^x + 1)^2 + 1$ we obtain in the interval [0,4] the following bifurcation diagram:



For the the horizontal value $a=1$ you can in a vertical cut read exactly the mentioned 8 values. This diagram can be interpreted as blur chart or/and as potential development chart.

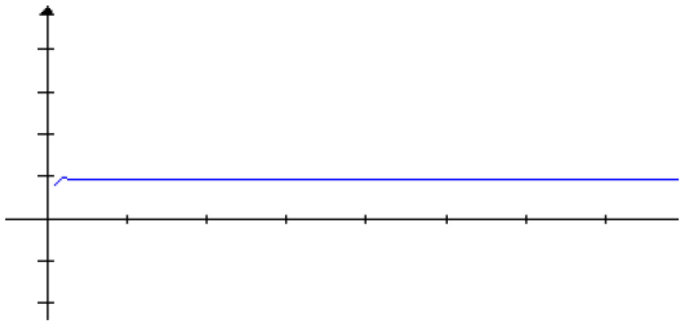
If you enlarged a slightly, so the system evolves in a bifurcation cascade, in which the limit cycle in ever shorter time grows exponentially to the so-called chaotic attractor. This means total disorientation in needs, confusion, which may be present e.g. in schizophrenic bursts. It is notable that this chaotic situation will with further increasing of the parameter a reverse, as you can see from the chart.

This are maybe indications that in this way psychological crises could get mathematically analyzable.

For another test person – an older woman – a very different picture emerged. The family of the recursions was rational:

$$n_{i+1}=\frac{72\cdot(\sqrt{3}+2)\cdot n_i\cdot(a\cdot n_i^2+1)}{a\cdot n_i^2\cdot(a\cdot n_i^2+578)+1}$$

For the original $a=1$ came out the following iteration graph, which has stabilised almost immediately at the point attractor 0,93. So a very balanced image.



If we vary again the parameter a in the interval [2,4], in the positive vicinity of $a=1$ the image remains very stable. If the negative values of a have any historical significance actually a wild chaos must have reigned there: